EFFECT OF PECLET NUMBER ON THE PRECIPITATION OF PARTICLES FROM A LAMINAR FLOW

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The problem of convective diffusion under steady-state laminar flow conditions is examined with allowance for diffusion in the direction of flow.

For cylindrical symmetry the equation of the process takes the form

$$\frac{\partial^2\psi}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial\psi}{\partial r}+\frac{\partial^2\psi}{\partial x^2}-2\,\frac{\bar{u}}{D}\left(1-\frac{r^2}{a^2}\right)\frac{\partial\psi}{\partial x}=0. \quad (1)$$

Going over to the variables $\varphi = \psi/\psi_0$, $\rho = r/a$, $h = x/a\,\mathrm{Pe}$, we obtain

$$\frac{\partial^2\varphi}{\partial\rho^2}+\frac{1}{\rho}\cdot\frac{\partial\varphi}{\partial\rho}+\mathrm{Pe}^{-2}\,\frac{\partial^2\varphi}{\partial h^2}-(1-\rho^2)\frac{\partial\varphi}{\partial h}=0$$

$$\left(0<\rho<1,\ 0<h<\infty,\ \varphi(\rho,\,0)=1,\ \varphi(1,\,h)=0\right). \quad (2)$$

This problem was solved in [1] for the case of large Peclet numbers. In [2] Eq. (1) was examined in the course of an investigation of the problem of increased vapor condensation on the walls of a gas duct; the author found an expression for the first root of the series expansion of the unknown function on the assumption that it is small. The results of an analysis of particle concentration on the channel axis are presented in [3].

In this article Eq. (2) is investigated with the object of finding the ratio of the particle fluxes in a certain section and at the sampler inlet, since it is precisely this quantity that most fully characterizes the precipitation rate.

We denote

$$\Psi=\int_0^\infty \varphi(\rho,\,h)\exp(-ph)\,dh,$$

$$\frac{\partial^2\Psi}{\partial\rho^2}+\frac{1}{\rho}\cdot\frac{\partial\Psi}{\partial\rho}+\left[p^2/\mathrm{Pe}^2+p(1-\rho^2)\right]\Psi=p/\mathrm{Pe}^2+(1-\rho^2)+\frac{1}{\mathrm{Pe}^2}\cdot\left.\frac{\partial\varphi}{\partial h}\right|_{h=0}. \quad (3)$$

From the boundary conditions and from the fact that the diffusion component of the particle flux at the inlet is a finite nonzero quantity, we have

$$\left.\frac{\partial\varphi}{\partial x}\right|_{x=0}=-\delta(a-r). \quad (4)$$

In accordance with [4], the fundamental system of Eqs. (3) comprises the functions

$$\Psi_1=\exp\left(-\frac{\lambda}{2}\,\rho^2\right)F\left(\frac{1}{2}-\frac{\lambda^3+\lambda\,\mathrm{Pe}^2}{4\mathrm{Pe}^2},\ 1,\ \lambda\rho^2\right),$$

$$\Psi_2=\exp\left(-\frac{\lambda}{2}\,\rho^2\right)U\left(\frac{1}{2}-\frac{\lambda^3+\lambda\,\mathrm{Pe}^2}{4\mathrm{Pe}^2},\ 1,\ \lambda\rho^2\right),$$

where $\lambda^2 = -p$; F and U are degenerate hypergeometric functions. As can be seen by inspection, a solution of Eq. (3) satisfying the condition $\Psi|_{\rho=1} = 0$ is given by the function

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TABLE 1. Numerical Values of the Roots and Coefficients of Eq. (12)

№	Pe	λ_0	A_0	λ_1	A_1	λ_2	A_2
1	0,01	0,15495	0,41576	0,23487	0,18083	0,29411	0,115315
2	0,04	0,30914	0,41559	0,46931	0,17985	0,58788	0,11460
3	0,1	0,48642	0,41531	0,74066	0,17795	0,92844	0,11319
4	0,3	0,82890	0,41505	1,27489	0,17199	1,60185	0,10880
5	0,75	1,26355	0,41747	1,98779	0,16046	2,51067	0,10027
6	1	1,42981	0,42028	2,27757	0,15500	2,88504	0,096199
7	2,9	2,09981	0,45951	3,65867	0,12708	4,73646	0,074904
8	6	2,45812	0,53371	4,79919	0,10645	6,42574	0,057664
9	8	2,54742	0,57203	5,23642	0,09999	7,15181	0,051388
10	10	2,59693	0,60251	5,54688	0,09610	7,71385	0,047012
11	14	2,64586	0,64647	5,94273	0,092222	8,51929	0,041449
12	20	2,67460	0,68738	6,24887	0,090495	9,25633	0,037094
13	30	2,69085	0,72494	6,46391	0,09058	9,88066	0,034017
14	40	2,69671	0,74588	6,55218	0,09132	10,1785	0,032816
15	50	2,69945	0,75922	6,59595	0,09205	10,3339	0,32287
16	70	2,70185	0,77522	6,63575	0,09318	10,4936	0,031919
17	100	2,70313	0,78777	6,65758	0,09420	10,5827	0,031846
18	200	2,70405	0,80305	6,67362	0,09573	10,6502	0,032019
Gormley and Kennedy [1]		2,7044	0,8191	6,6790	0,0975	10,673	0,0325

$$\Psi(\rho,\ p)=\frac{1}{p}\left[1-\frac{\Psi_1(\rho,\ p)}{\Psi_1(1,\ p)}\right], \tag{5}$$

$$\varphi=\frac{1}{2\pi i}\int\limits_{c-i\infty}^{c+i\infty}\Psi(\rho,\ p)\exp(ph)\,dp=-2\sum_i\frac{\exp\left(-\frac{\lambda_i}{2}\rho^2\right)F(\rho,\ \lambda_i)}{\lambda_i\frac{\partial}{\partial\lambda}\Psi_1\Big|_{\substack{\rho=1\\ \lambda=\lambda_i}}}\exp(-\lambda_i^2h), \tag{6}$$

where λ_i are the roots of the equation $\Psi_1(1,\ \lambda)=0$. The expression for the particle flux takes the form

$$\mu(h)=4\int\limits_0^1\varphi(\rho,\ h)(1-\rho^2)\rho d\rho-\frac{4}{\mathrm{Pe}^2}\int\limits_0^1\frac{\partial\varphi}{\partial h}\rho d\rho, \tag{7}$$

$$\mu(0)=1+4/\mathrm{Pe}. \tag{8}$$

Thus, we obtain

$$n(h)=8\left(1+\frac{4}{\mathrm{Pe}}\right)^{-1}\sum_{i=0}^{\infty}\frac{\Psi_1'(1,\ \lambda_i)}{\lambda_i^3(\Psi_1)_\lambda'\Big|_{\substack{\rho=1\\ \lambda=\lambda_i}}}\exp(-\lambda_i^2h). \tag{9}$$

It is easy to show that the equation for the eigenvalues has only real roots.

At Pe $\leqslant$ 1 we obtain approximate values of the roots from the relation

$$\Psi_1(\rho,\ \lambda)\to I_0(p/\mathrm{Pe}\,\rho)\quad\text{as}\quad\rho\to 1; \tag{10}$$

when Pe > 1 the approximations for the roots follow from [1]

$$\lambda_i\cong 2.67+4i,\ i=0,\ 1,\ 2,\ 3,\ \ldots \tag{11}$$

The values of the roots were next refined by Newton's method on a computer. Taking the first three terms in (9), we write the expression for n(h) in the form

$$n(h)=A_0\exp(-\lambda_0^2h)+A_1\exp(-\lambda_1^2h)+A_2\exp(-\lambda_2^2h)+\ldots \tag{12}$$

The numerical values of A_i and λ_i in this expression are given in Table 1 for a series of values of the Peclet number.

The plane problem of convective diffusion, which describes particle precipitation in a so-called plane-parallel diffusion battery, is also of practical importance. As before, we obtain

$$n'(h)=3\left(1+\frac{2}{\mathrm{Pe}}\right)^{-1}\sum_{i=0}^{\infty}\frac{\Phi'(1,\ \lambda_i)\exp(-\lambda_i^2h)}{\lambda_i^3\frac{\partial}{\partial\lambda}\Phi(1,\ \lambda)\big|_{\lambda=\lambda_i}}=\sum_{i=0}^{\infty}A_i'\exp(-\lambda_i^2h), \tag{13}$$

TABLE 2. Numerical Values of the Roots and Coefficients of Eq. (13)

№	$Pe\cdot\frac{3}{4}$	λ_0	A_0	λ_1	A_1	λ_2	A_2
1	0,01	0,12516	0,63538	0,21700	0,21097	0,28019	0,12654
2	0,05	0,27832	0,63072	0,48452	0,20619	0,62598	0,12349
3	0,1	0,39089	0,62547	0,68396	0,20055	0,88433	0,11990
4	0,5	0,82712	0,60108	1,50721	0,16564	1,96053	0,09762
5	1	1,09300	0,59714	2,09303	0,13788	2,74313	0,07980
6	2	1,36006	0,62331	2,85460	0,10656	3,79773	0,05946
7	10	1,65587	0,79610	4,89118	0,05822	7,21612	0,02458
8	16	1,67122	0,83405	5,27389	0,05402	8,17506	0,01988
9	20	1,67490	0,84794	5,39549	0,05306	8,54908	0,01840
10	50	1,68051	0,88402	5,61930	0,05222	9,41693	0,01566
11	100	1,68132	0,89694	5,65692	0,05252	9,60103	0,01527
12	400	1,68158	0,90695	5,66904	0,05295	9,66394	0,01524
13	$\to\infty$	1,68159	0,91035	5,66986	0,05314	9,66824	0,01528

$$\Phi(\rho,\ \lambda) = \exp\left(-\frac{\lambda}{2}\rho\right) F\left(\frac{1}{4} - \frac{\lambda^3 + \lambda\,Pe^2\frac{9}{16}}{4Pe^2\frac{9}{16}}\ \frac{1}{2},\ \lambda\rho^2\right),$$

$$h = \frac{4}{3}\cdot\frac{x'}{b\,Pe},$$

$$\Phi(1,\ \lambda_i) = 0,\ i = 0,\ 1,\ 2,\ \ldots,\ \rho = y/b.$$

Certain values of A_i' and λ_i in (13) are presented in Table 2. Calculations based on (9) and (13) show that the effect of diffusion in the direction of flow can be neglected at Pe $\geqslant$ 50.

NOTATION

ψ	is the concentration of diffusing particles;
ψ_0	is the entrance particle concentration;
a	is the tube radius;
b	is the half-distance between planes;
D	is the particle diffusion coefficient, cm^2/sec;
φ	is the particle concentration referred to the entrance concentration;
$\bar{u}$	is the mean flow velocity;
Pe	is the Peclet number, $Pe = d\bar{u}/D$, d = 2a for cylindrical geometry and d = 2b for rectangular geometry;
r and x	are cylindrical coordinates;
y and x'	are Cartesian coordinates.

LITERATURE CITED

1. P. G. Gormley and Kennedy, Proc. Roy. Irish Acad., 52, Section A, No. 12 (1949).
2. G. M. Filippov, Izv. Akad. Nauk SSSR, Mekhan. Zhidk. i Gaz., No. 4 (1967).
3. S. Jarvis Jr., J. Res. NBS, 72B, No. 2, 135 (1968).
4. E. Madelung, Mathematical Tools for the Physicist, Dover, New York (1943).